

How Engaging Puzzles Build Algebraic Habits of Mind

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Solving Who Am I? Puzzles

Who Am I?

- The product of my digits is 7.
- The sum of my digits is 8.
- My units digit is greater than my tens digit.

t	u
<input type="text"/>	<input type="text"/>

Who Am I?

- The product of my digits is 16.
- The sum of my digits is 8.

t	u
<input type="text"/>	<input type="text"/>

Who Am I?

- I am even.
- My tens digit is 1.
- The product of my three digits is 12.
- h is four less than my units digit.

h	t	u
<input type="text"/>	<input type="text"/>	<input type="text"/>

Who Am I?

- I am a multiple of 10.
- My hundreds digit is one more than my tens digit.
- The sum of my three digits is 7.

h	t	u
<input type="text"/>	<input type="text"/>	<input type="text"/>

Who Am I?

- I am odd.
- $u > t$
- My hundreds digit is prime.
- $t = 2h$
- Two of my digits are square numbers.

h	t	u
<input type="text"/>	<input type="text"/>	<input type="text"/>

Who Am I?

- I am in the thirties.
- $d < t$
- My units digit is twice my tens digit.
- d is four less than u .
- The product of d and t is u .

t	u	d
<input type="text"/>	<input type="text"/>	<input type="text"/>

Who Am I?

- t is a perfect square.
- $h - u = 0$
- The sum of h and u is 13 more than t .
- I am divisible by 3.

h	t	u
<input type="text"/>	<input type="text"/>	<input type="text"/>

Who Am I?

- $u - h = t$
- $3t = h$
- All of my digits are not odd.
- Not all of my digits are odd.
- Not all of my digits are divisible by 3.

h	t	u
<input type="text"/>	<input type="text"/>	<input type="text"/>



transition to algebra

make algebra make sense

Curriculum information and presentation documents: ttalgebra.edc.org

Sample materials and ordering information: transitiontoalgebra.com



SolveMe Mobiles: solve.me and coming soon to iTunes app store



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Who Am I?

n	k	h	t	u

- $t < u$
- $h + k = u$
- I am a multiple of 5.
- k and h are square numbers.
- My only even digit is k .
- I'm less than 75432.
- No two of my digits are the same.
- $t + k = n$

Who Am I?

h	t	u

- $u^2 = h + t$
- $h + t = 2u$
- $h > t$
- $ht > u$

Who Am I?

k	h	t	u

- No two of my digits are the same.
- Three of my digits are powers of 2.
- At least one of my digits is prime.
- The sum of my digits is a perfect square.
- $h + t = k + 2u$
- Three of my digits are perfect squares.
- The difference between t and u is 5.
- $9h = t$

Who Am I?

h	t	u

- I'm odd.
- $h + t + u = 6$
- My units digit $>$ my tens digit.
- $h > t$
- $t + u = h$
- The product of my three digits is 0.

Who Am I?

u	d	c	m

- I am less than 1.63.
- I am greater than 1.6.
- c is one-third of d .
- $m > c^3$

Building Who Am I? Puzzles

Puzzle Building Steps:

1. Choose the final answer & construct clue boxes.
2. Create clues to help identify the answer.
3. Check that the clues lead to a unique solution.

n	k	h	t	u	d	c	m

These variables were selected to match place values and metric system prefixes (n for thousands, k for kilo-, h for hundreds, t for tens, u for units, d for deci-, c for centi-, and m for mili-).

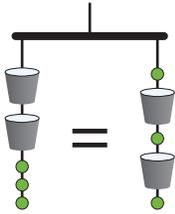
Who Am I?

Who Am I?

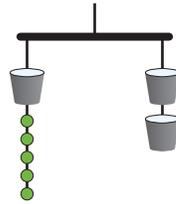
Solving Mobile Puzzles

In each of these problems, a dot (●) equals 1.

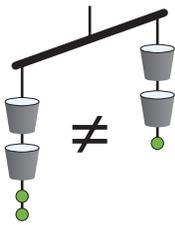
① This mobile *always balances*. Why?



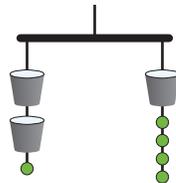
② This mobile *only balances* when the buckets represent a certain number. What number makes it balance?



③ This mobile *never balances* no matter what number the bucket represents. Why?



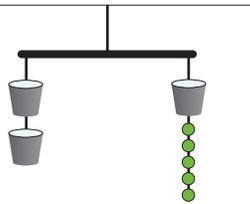
④ Does this mobile balance *always, sometimes, or never*?



If sometimes, *when*?

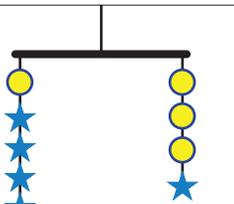
Every beam in the mobiles below is balanced. The strings and the beams weigh nothing. Find the weight of each shape.

⑤



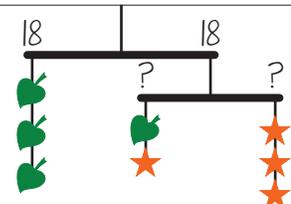
● = 1 ☪ = _____

⑥



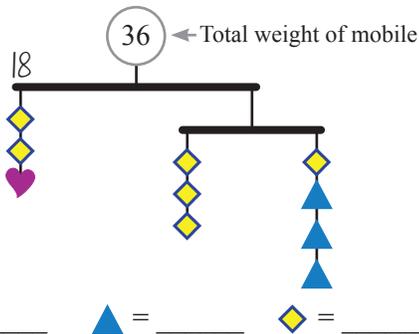
★ = 4 ● = _____

⑦



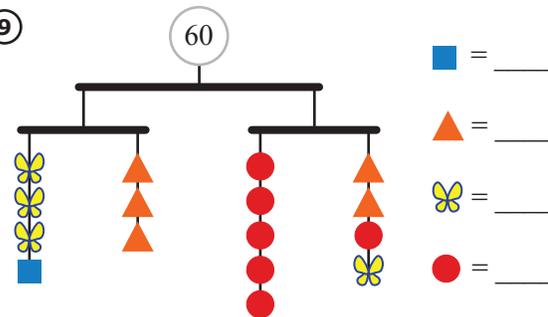
🍃 = 6 ★ = _____

⑧



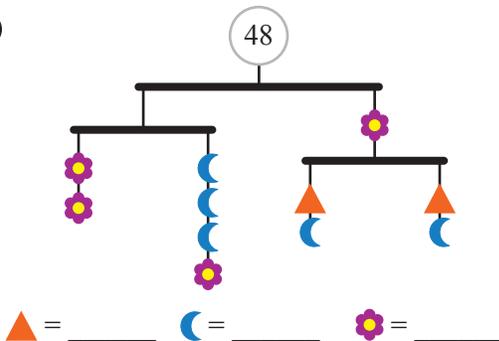
♥ = _____ ▲ = _____ ◆ = _____

⑨



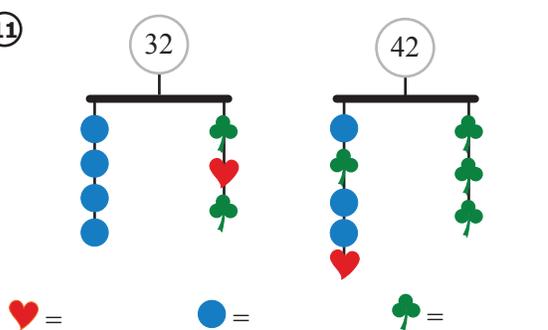
■ = _____
▲ = _____
🦋 = _____
● = _____

⑩



▲ = _____ ☾ = _____ 🌸 = _____

⑪



♥ = _____ ● = _____ ♣ = _____

Building Mobile Puzzles

To make a mobile puzzle, start by picking your own shapes and making up the solutions first. Then make up a balanced mobile.

= _____
 = _____
 = _____

Before you share your mobile, make sure that the solutions you started with are the only possible solutions.

Mystery Number Puzzles

- ① What are the only two numbers that  could be if $\text{yellow circle} \cdot \text{yellow circle} = \text{yellow circle}$?

 = ____ or ____

- ② What is the only number that  could be if $\text{red heart} + \text{red heart} = \text{red heart}$?

 = ____

- ③ What could , , and  be if all the variables represent different numbers?

$$\begin{array}{l} \text{blue water drop} \cdot \text{red hexagon} = \text{blue star} \\ \text{red hexagon} + \text{red hexagon} = \text{blue star} \\ \text{blue water drop} + \text{blue water drop} + \text{blue water drop} = \text{blue star} \end{array} \quad \begin{array}{l} \text{blue star} = \underline{\hspace{2cm}} \\ \text{blue water drop} = \underline{\hspace{2cm}} \\ \text{red hexagon} = \underline{\hspace{2cm}} \end{array}$$

- ④ What could  and  be?

$$\begin{array}{l} \text{yellow pentagon} \cdot \text{yellow pentagon} = \text{green clover} \\ \text{yellow pentagon} + \text{yellow pentagon} = \text{green clover} \\ \text{yellow pentagon} \neq \text{green clover} \end{array} \quad \begin{array}{l} \text{yellow pentagon} = \underline{\hspace{2cm}} \\ \text{green clover} = \underline{\hspace{2cm}} \end{array}$$

- ⑤ What could , , , , and  be if all the shapes are *different* single-digit numbers (0-9)?

$$\begin{array}{l} \text{red square} \cdot \text{red square} = \text{red square} \\ \text{red square} + \text{purple triangle} = \text{green diamond} \\ \text{blue circle} + \text{blue circle} = \text{green diamond} \\ \text{red square} + \text{yellow pentagon} = \text{blue circle} \\ \text{yellow pentagon} \cdot \text{blue circle} = \text{green diamond} \end{array} \quad \begin{array}{l} \text{red square} = \underline{\hspace{2cm}} \\ \text{yellow pentagon} = \underline{\hspace{2cm}} \\ \text{green diamond} = \underline{\hspace{2cm}} \\ \text{blue circle} = \underline{\hspace{2cm}} \\ \text{purple triangle} = \underline{\hspace{2cm}} \end{array}$$

- ⑥ What could , , , , and  be if all the shapes are *different* single-digit numbers (0-9)?

$$\begin{array}{l} \text{blue pentagon} \cdot \text{yellow triangle} = \text{purple diamond} \\ \text{yellow triangle} + \text{yellow triangle} = \text{purple diamond} \\ \text{blue pentagon} + \text{blue pentagon} = \text{yellow triangle} \\ \text{blue pentagon} + \text{yellow triangle} = \text{red star} \\ \text{blue pentagon} \cdot \text{green circle} = \text{red star} \end{array} \quad \begin{array}{l} \text{blue pentagon} = \underline{\hspace{2cm}} \\ \text{purple diamond} = \underline{\hspace{2cm}} \\ \text{green circle} = \underline{\hspace{2cm}} \\ \text{yellow triangle} = \underline{\hspace{2cm}} \\ \text{red star} = \underline{\hspace{2cm}} \end{array}$$

- ⑦ None of , , , , and  are negative. Use the clues below to figure out the values.

$$\begin{array}{l} \text{red triangle} \cdot \text{purple pentagon} = \text{purple pentagon} \\ \text{yellow square} \cdot \text{yellow square} = \text{yellow square} \\ \text{green circle} + \text{yellow square} = \text{red triangle} \cdot \text{red triangle} \\ \text{blue star} + \text{blue star} = \text{red triangle} \\ \text{blue star} \cdot \text{blue star} = \text{red triangle} \\ \text{yellow square} > \text{purple pentagon} \end{array} \quad \begin{array}{l} \text{red triangle} = \underline{\hspace{2cm}} \\ \text{yellow square} = \underline{\hspace{2cm}} \\ \text{green circle} = \underline{\hspace{2cm}} \\ \text{blue star} = \underline{\hspace{2cm}} \\ \text{purple pentagon} = \underline{\hspace{2cm}} \end{array}$$

- ⑧ , , , , and  can be any numbers at all and $\text{yellow circle} \neq \text{red heart}$. Figure out what they must be.

$$\begin{array}{l} \text{red heart} \cdot \text{red heart} = \text{red heart} \\ \text{red heart} + \text{red heart} = \text{red heart} \\ \text{green apple} \cdot \text{yellow circle} = \text{blue butterfly} \cdot \text{purple square} \\ \text{green apple} + \text{yellow circle} = \text{red heart} \\ \text{yellow circle} \cdot \text{yellow circle} = \text{purple square} \\ \text{yellow circle} + \text{yellow circle} = \text{purple square} \end{array} \quad \begin{array}{l} \text{blue butterfly} = \underline{\hspace{2cm}} \\ \text{purple square} = \underline{\hspace{2cm}} \\ \text{green apple} = \underline{\hspace{2cm}} \\ \text{yellow circle} = \underline{\hspace{2cm}} \\ \text{red heart} = \underline{\hspace{2cm}} \end{array}$$

Solving MysteryGrid Puzzles

Use the clues to fill in each grid so that every row and every column contains all of the numbers in the title.

5, 7, 9 Latin Square

	5	
7	9	
		9

r, s, t Latin Square

	<i>s</i>	
		<i>r</i>

MysteryGrid **3, 4, 5**

2, -	$\frac{3}{5}$	7, +
20, •	4	
	8, +	

MysteryGrid **3, 6, 9**

18, •		15, +
54, •	15, +	

MysteryGrid **3, 4, 5, 6**

11, +		36, •	
4	15, •	10, +	
18, •			30, •
	20, •		

MysteryGrid $\frac{1}{3}, \frac{1}{2}, 1$

$1\frac{1}{2}, +$	$\frac{1}{6}, •$	
	$\frac{1}{2}, -$	$\frac{1}{3}, •$
$\frac{1}{3}$		

MysteryGrid **6, 7, 8, 9**

30, +	72, •	63, •	
		30, +	
		48, •	
42, •			

MysteryGrid **1, 2, 3, 4**

8, •		6, •	4, •
4, +			
3, -	5, +	7, +	
		3, +	

MysteryGrid **1, 3, 5, 7**

21, •	7, •		15, +
	25, •		
2, -			
12, +		3, ÷	

MysteryGrid **0.1, 0.2, 0.3, 0.4**

.6, +		.08, ×	
	.016, ×	3, ÷	
.12, ×			.5, +
	.02, ×		

MysteryGrid **a, a², a³**

$a^4, •$	$a^2 + a, +$	
	$a^6, •$	$a^5, •$

MysteryGrid **(a - 1), a, (a + 1)**

$2a - 1, +$		$a^2 + a, •$
$2a + 1, +$	$a - 1$	
	$a^2 - 1, •$	

MysteryGrid $(c + 3), (c + 4), (c + 5)$

$c^2 + 8c + 15, \bullet$		$c^2 + 7c + 12, \bullet$
$c^2 + 9c + 20, \bullet$	$2c + 7, +$	
		$c + 5$

MysteryGrid a^{-1}, a, a^2, a^3

a^3, \bullet	a, \bullet	a^6, \bullet	
a^4, \bullet		a^5, \bullet	$1, \bullet$
a, \bullet			

MysteryGrid $2, 3, 5, 7, 11$

$42, \bullet$		3	$35, \bullet$	$110, \bullet$
	$55, \bullet$			
$27, +$	7	$12, \bullet$		
		$23, +$		$21, \bullet$
$6, \bullet$				

Building MysteryGrid Puzzles

- Choose a grid size and pick a combination of three or four numbers or expressions with variables. Fill in the grid like a Latin Square puzzle with exactly one of each number or expression in each row and column.

- Then make your cages. Block off a group of numbers. Use an operation (+, -, •, or ÷) to make your clue. These example grids have been started.



MysteryGrid $2, 3, 4, 7$

2	7	$9, +$ 4	3
3	4	7	2
7	2	3	4
4	3	$5, -$ 2	7

MysteryGrid $2x, (x + 2), x^2$

$2x^3, \bullet$ 2x	x^2	$x + 2$
x^2	$x + 2$	2x
$x + 2$	2x	x^2

You can make cages with just one number, too.

For subtraction and division, use cages with only two numbers.

- Make sure there is *only one solution*.

Puzzles with more than one solution aren't "wrong," but they aren't satisfying because the player will get stuck at the point where there is no unique answer.

Copy only your clues and try solving your puzzle yourself before sharing it with someone else.

Adjust the cages as needed to make the puzzle have only one solution.

MysteryGrid $_, _, _$

MysteryGrid $_, _, _, _$
