



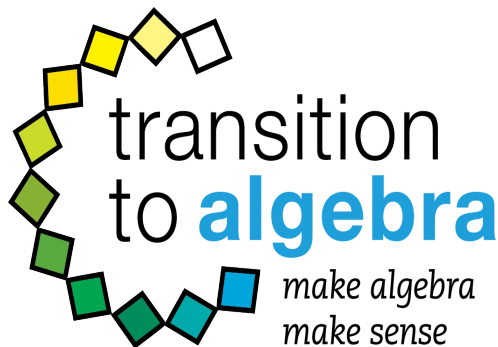
How Engaging Puzzles Build Algebraic Habits of Mind

Kentucky Center for Mathematics Conference (Lexington, KY)

March 10, 2015

Jane M. Kang (jkang@edc.org)

Transition to Algebra



- A coherent, full-year NSF-funded algebra support curriculum organized around five key ***algebraic habits of mind***
- Quickly giving students the mathematical knowledge, skill, and confidence to succeed in a first-year algebra class

Algebraic Habits of Mind

- Puzzling and Persevering
- Seeking and Using Structure
- Using Tools Strategically
- Describing Repeated Reasoning
- Communicating with Precision

I am a 3-digit number

Who Am I?

- I am even.
- My digits are all different.
- I am greater than 319.
- My hundreds digit is less than 7.
- $u = 1 + h$
- My tens digit is my largest digit.
- My hundreds digit is my only odd digit.
- My units digit is one more than my hundreds digit.
- The sum of all three of my digits is 19.
- My units digit is not 4.

<i>h</i>	<i>t</i>	<i>u</i>

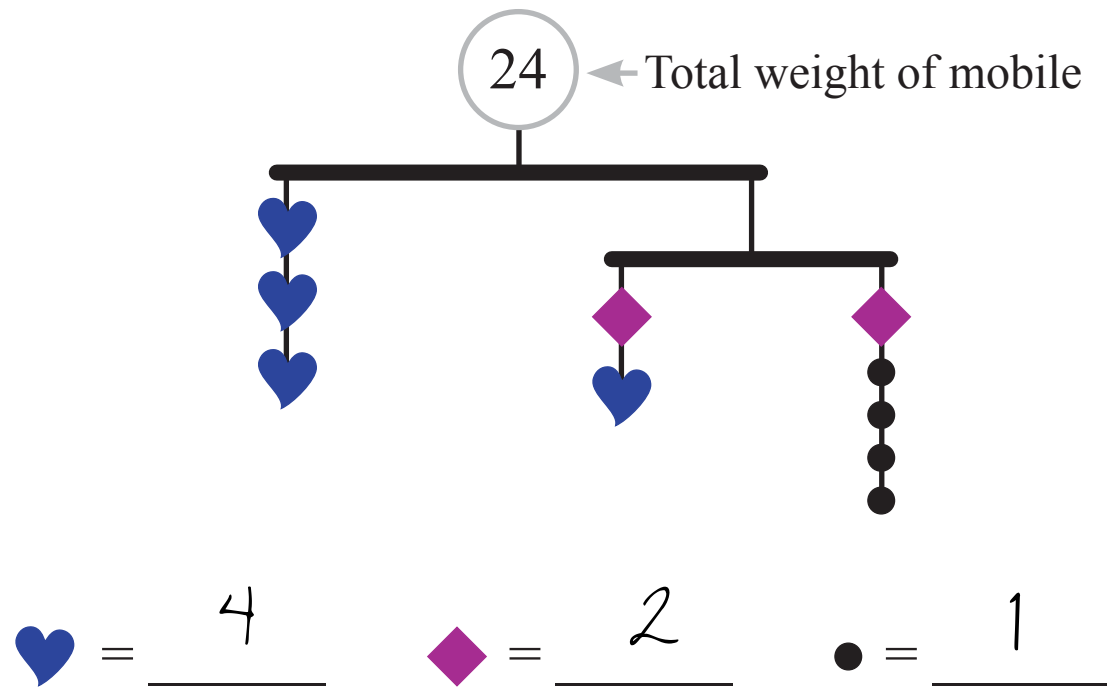
Who Am I? Puzzles

Who Am I?

- I am less than $\frac{1}{2}$. $\frac{\text{numerator}}{\text{denominator}} = \frac{\boxed{}}{\boxed{}}$
- I am not 0.
- My numerator and my denominator both have one digit.
- My denominator is 3 more than my numerator.
- Neither my numerator nor my denominator is a square number.

Find the coefficients a , b , and c in the quadratic $y = ax^2 + bx + c$ so that the roots are $-\frac{2}{3}$ and $-\frac{1}{2}$ and the minimum value y can attain is $8\frac{1}{8}$.

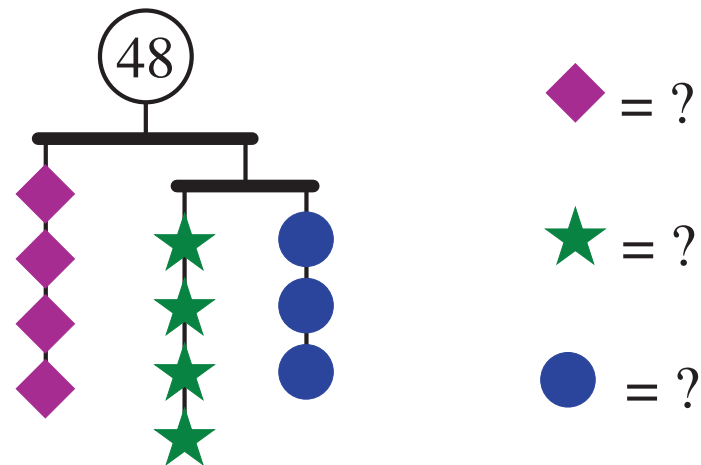
Mobile Puzzles



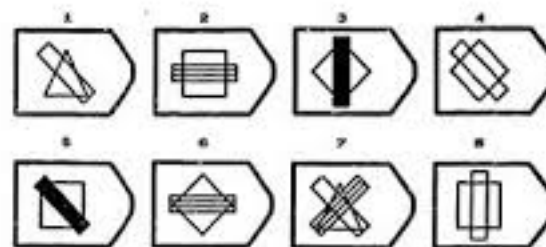
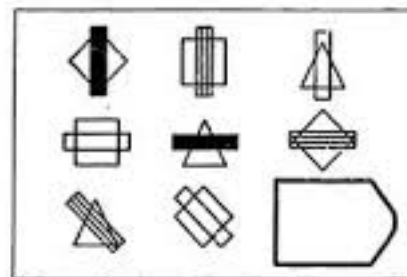
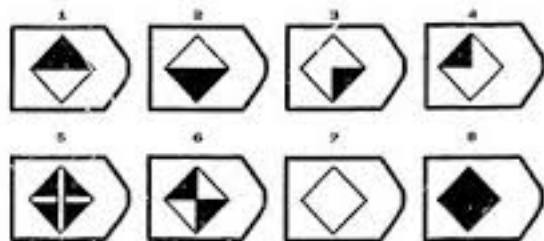
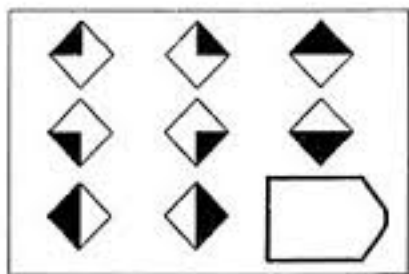
Why Puzzles?

Mathematical Puzzles:

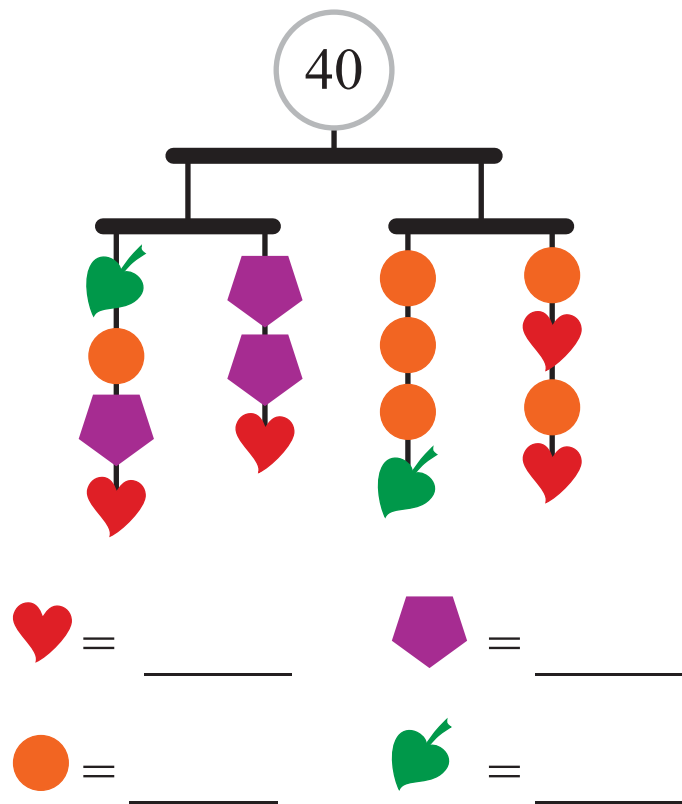
- (are **fun** and **engaging**)
- emphasize the logic of algebra
- help students develop strategy in problem solving
- remove the stigma associated with not knowing how to solve a math problem



Raven's Progressive Matrices



Mobile Puzzles



$$\text{apple} + \heartsuit + \text{pentagon} + \text{circle} = \text{pentagon} + \text{pentagon} + \heartsuit$$

$$\text{apple} + \text{circle} = \text{pentagon}$$

$$\text{circle} + \text{circle} + \text{circle} + \text{apple} = \text{circle} + \heartsuit + \text{circle} + \heartsuit$$

$$\text{circle} + \text{apple} = \heartsuit + \heartsuit$$

$$\heartsuit + \heartsuit = \text{pentagon}$$

Mystery Number Puzzles

③ What could , , and  be if all the variables represent different numbers?

$$\text{drop} \cdot \text{hexagon} = \text{star}$$

$$\text{hexagon} + \text{hexagon} = \text{star}$$

$$\text{drop} + \text{drop} + \text{drop} = \text{star}$$

$$\text{star} = \underline{\hspace{2cm}}$$

$$\text{drop} = \underline{\hspace{2cm}}$$

$$\text{hexagon} = \underline{\hspace{2cm}}$$

Mystery Number Puzzles

$$\text{red pentagon} + \text{red pentagon} = \text{red pentagon}$$

Only one solution

$$\text{yellow hexagon} \cdot \text{yellow hexagon} = \text{yellow hexagon}$$

Two solutions

$$\text{purple triangle} \cdot \text{blue square} = \text{purple triangle}$$

For some value of ,  can have any value.
For some value of ,  can have any value.

$$\text{purple diamond} + \text{purple diamond} = \text{orange circle}$$

$$\text{purple diamond} \cdot \text{purple diamond} = \text{orange circle}$$

Two solutions (assuming different variables can have the same value)

What makes a *good* puzzle?

- Puzzles are the “main course”
 - Mathematical in both ***content*** and ***nature***
- Two dimensions: Cognitive challenge & Required arithmetic/mathematical knowledge
- Good puzzles *feel* do-able and are challenging

Latin Squares Puzzles

- Use the clues to fill in the grid so that every row and every column contains one of each element.

a*, *b*, *c Latin Square

<i>c</i>		<i>a</i>
	<i>c</i>	

MysteryGrid Puzzles

- In MysteryGrid puzzles, the numbers in each “cage” should reach the target number using the given operation.
- For example, a 3-cell, “20, \times ” cage means you need to fill that cage with 3 numbers that multiply to 20.

MysteryGrid 1, 3, 4, 5

4, +		4, \div	1, -
20, \times	12, +		
			2, -
	15, \times		

MysteryGrid Puzzles

MysteryGrid $x, 2x, 4x$

$8x^2, \bullet$	$3x, -$	
	$5x, +$	
$4x^2, \bullet$		

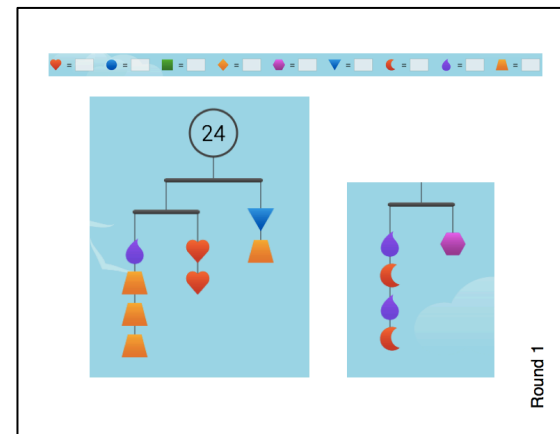
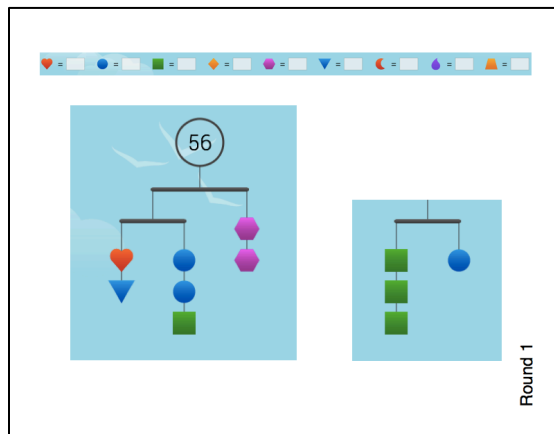
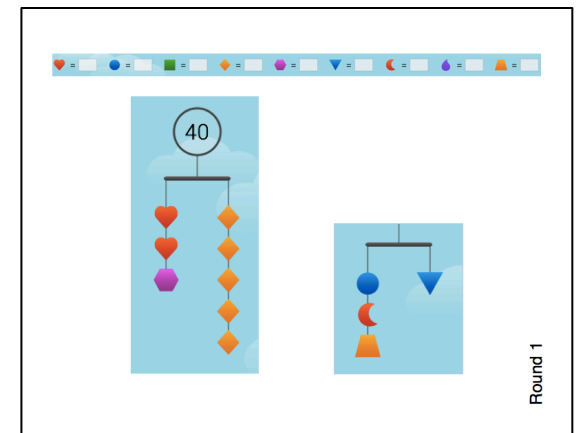
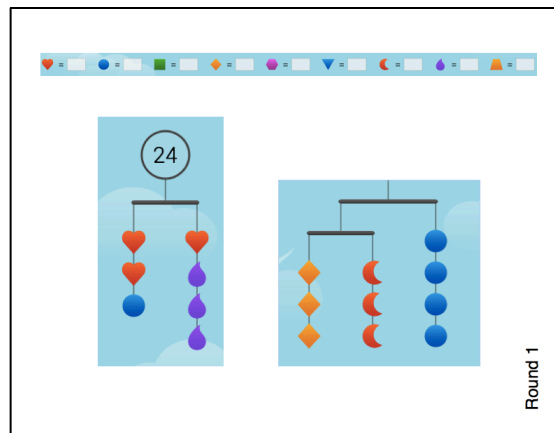
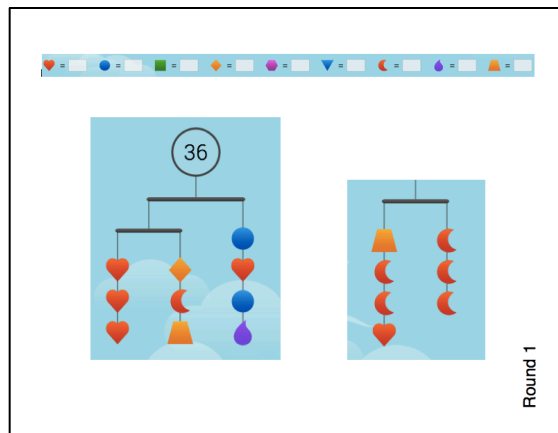
MysteryGrid a, a^2, a^3, a^4

a^6, \bullet			$2a^4+a^3, +$
a^7, \bullet	a^4, \bullet		
		a^5, \bullet	
	a^7, \bullet		

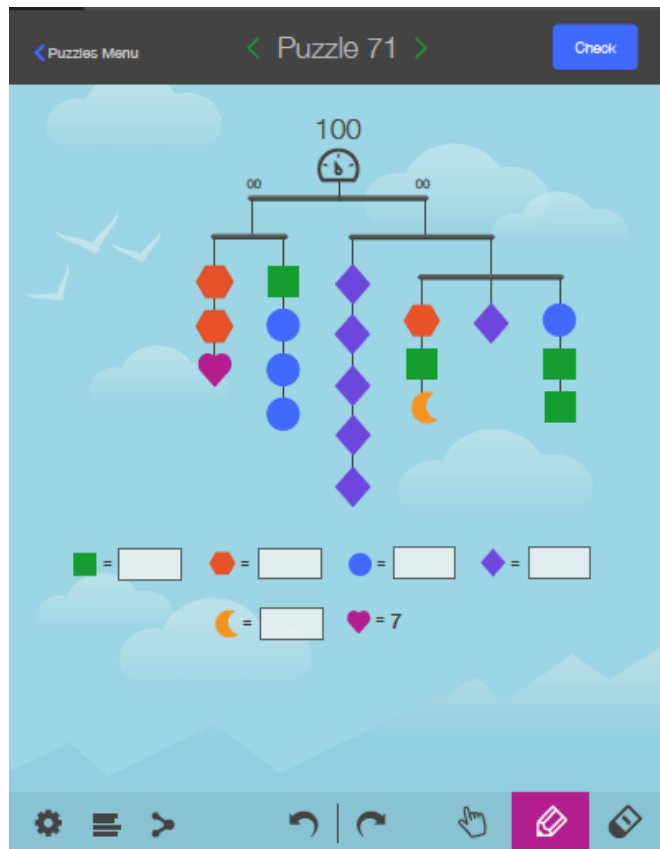
Puzzling and Persevering

- The I-can-puzzle-it-out disposition
 - Looking for an entry point
 - Building working memory
 - Depending on their own logic
- Puzzle creation: Being *producers*—not just consumers—of mathematics

Mobiles: Collaborative Game



Mobile Puzzle App: SolveMe



Interactive puzzling features:

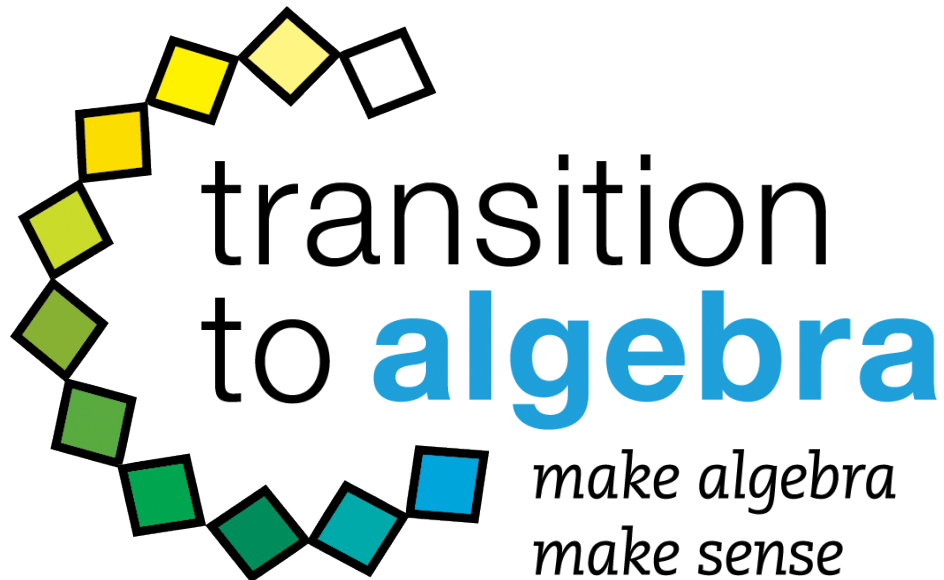
- Shape equations
- Subtraction, division, factoring
- Substitution
- Annotations
- Sharing
- “Build Your Own” mode

SolveMe.edc.org

Algebraic Habits of Mind

- Puzzling and Persevering
- Seeking and Using Structure
- Using Tools Strategically
- Describing Repeated Reasoning
- Communicating with Precision

Transition to Algebra



Full-year algebra-support curriculum with student & teacher materials that supports the Common Core Standards for Mathematical Practice

For for information: transitiontoalgebra.com or jkang@edc.org