



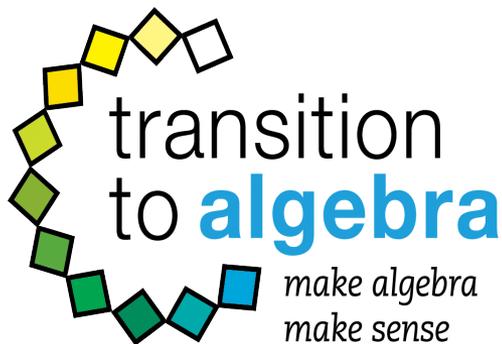
# How Engaging Puzzles Build Algebraic Habits of Mind

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# Transition to Algebra



- A coherent, full-year NSF-funded algebra support curriculum organized around five key ***algebraic habits of mind***
- Quickly giving students the mathematical knowledge, skill, and confidence to succeed in a first-year algebra class

# Algebraic Habits of Mind

- Puzzling and Persevering
- Seeking and Using Structure
- Using Tools Strategically
- Describing Repeated Reasoning
- Communicating with Precision

# I am a 3-digit number

Who Am I?

- I am even.
- My digits are all different.
- I am greater than 319.
- My hundreds digit is less than 7.
- $u = 1 + h$
- My tens digit is my largest digit.
- My hundreds digit is my only odd digit.
- My units digit is one more than my hundreds digit.
- The sum of all three of my digits is 19.
- My units digit is not 4.

<i>h</i>	<i>t</i>	<i>u</i>

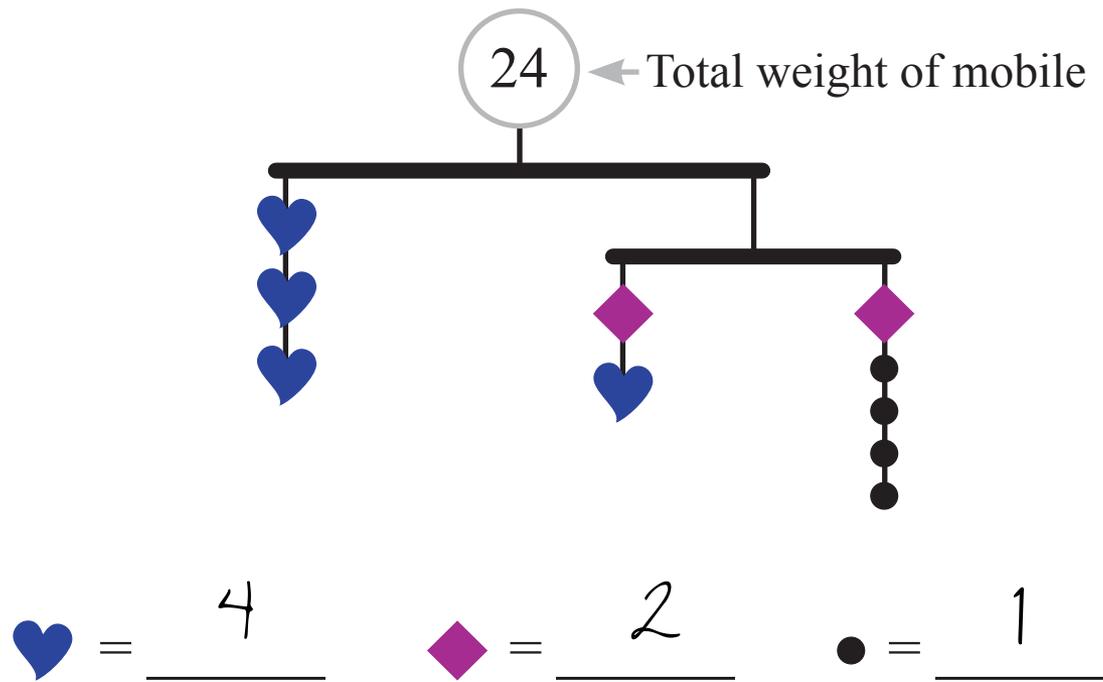
# Who Am I? Puzzles

## Who Am I?

- I am less than  $\frac{1}{2}$ .  $\frac{\text{numerator}}{\text{denominator}} = \frac{\boxed{\phantom{00}}}{\boxed{\phantom{00}}}$
- I am not 0.
- My numerator and my denominator both have one digit.
- My denominator is 3 more than my numerator.
- Neither my numerator nor my denominator is a square number.

Find the coefficients  $a$ ,  $b$ , and  $c$  in the quadratic  $y = ax^2 + bx + c$  so that the roots are  $-\frac{2}{3}$  and  $-\frac{1}{2}$  and the minimum value  $y$  can attain is  $8\frac{1}{8}$ .

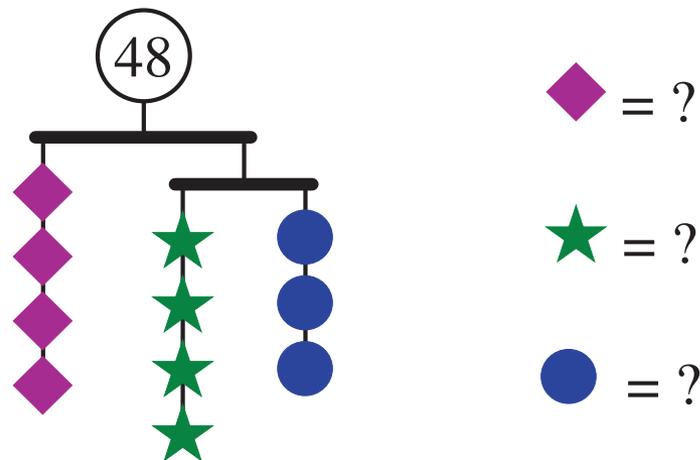
# Mobile Puzzles



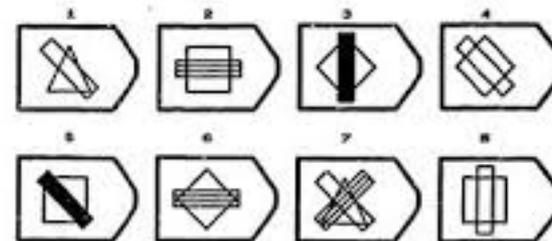
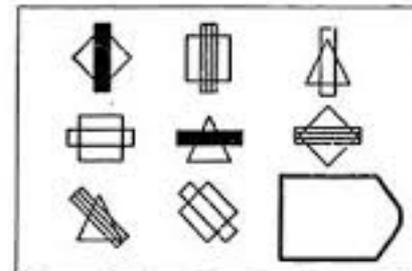
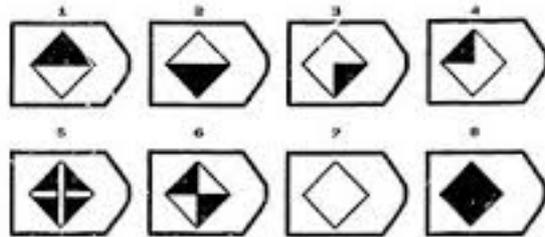
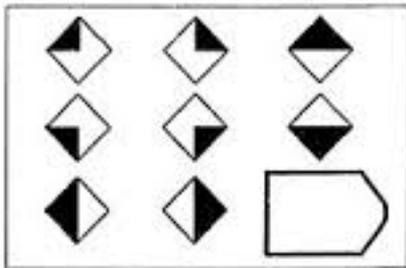
# Why Puzzles?

## Mathematical Puzzles:

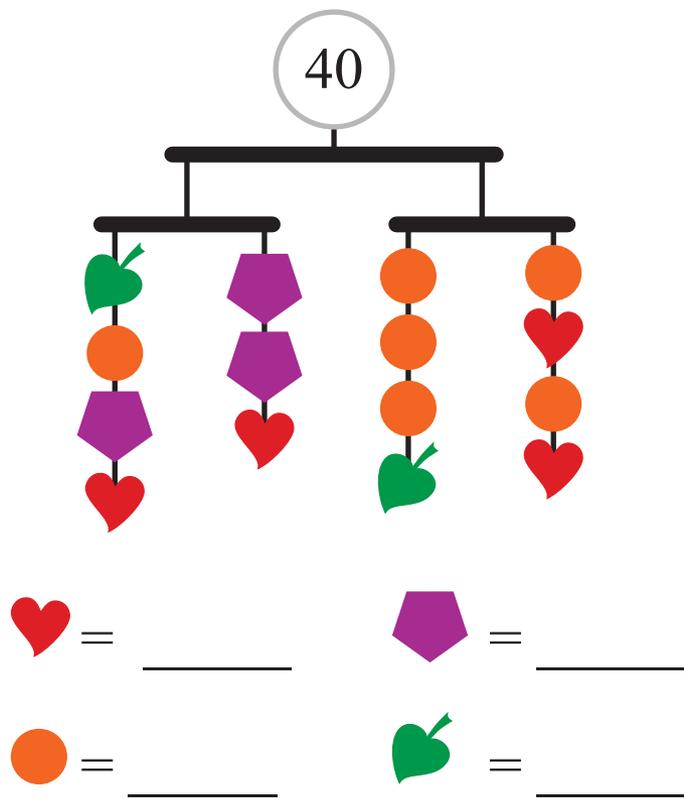
- (are **fun** and **engaging**)
- emphasize the logic of algebra
- help students develop strategy in problem solving
- remove the stigma associated with not knowing how to solve a math problem



# Raven's Progressive Matrices



# Mobile Puzzles



$$\heartsuit + \heartsuit + \blacklozenge + \bullet = \blacklozenge + \blacklozenge + \heartsuit$$

$$\heartsuit + \bullet = \blacklozenge$$

$$\bullet + \bullet + \bullet + \heartsuit = \bullet + \heartsuit + \bullet + \heartsuit$$

$$\bullet + \heartsuit = \heartsuit + \heartsuit$$

$$\heartsuit + \heartsuit = \blacklozenge$$

# Mystery Number Puzzles

③ What could , , and  be if all the variables represent different numbers?

$$\text{Water Drop} \cdot \text{Hexagon} = \text{Star}$$

$$\text{Hexagon} + \text{Hexagon} = \text{Star}$$

$$\text{Water Drop} + \text{Water Drop} + \text{Water Drop} = \text{Star}$$

$$\text{Star} = \underline{\hspace{2cm}}$$

$$\text{Water Drop} = \underline{\hspace{2cm}}$$

$$\text{Hexagon} = \underline{\hspace{2cm}}$$

# Mystery Number Puzzles

$$\color{red}\blacklozenge + \color{red}\blacklozenge = \color{red}\blacklozenge$$

Only one solution

$$\color{yellow}\blackhexagon \cdot \color{yellow}\blackhexagon = \color{yellow}\blackhexagon$$

Two solutions

$$\color{purple}\blacktriangle \cdot \color{cyan}\blacksquare = \color{purple}\blacktriangle$$

For some value of  $\color{purple}\blacktriangle$ ,  $\color{cyan}\blacksquare$  can have any value.  
For some value of  $\color{cyan}\blacksquare$ ,  $\color{purple}\blacktriangle$  can have any value.

$$\color{purple}\blacklozenge + \color{purple}\blacklozenge = \color{orange}\bullet$$

$$\color{purple}\blacklozenge \cdot \color{purple}\blacklozenge = \color{orange}\bullet$$

Two solutions (assuming different variables can have the same value)

# What makes a *good* puzzle?

- Puzzles are the “main course”
  - Mathematical in both ***content*** and ***nature***
- Two dimensions: Cognitive challenge & Required arithmetic/mathematical knowledge
- Good puzzles *feel* do-able and are challenging

# Latin Squares Puzzles

- Use the clues to fill in the grid so that every row and every column contains one of each element.

***a*, *b*, *c*** Latin Square

<i>c</i>		<i>a</i>
	<i>c</i>	

# MysteryGrid Puzzles

- In MysteryGrid puzzles, the numbers in each “cage” should reach the target number using the given operation.
- For example, a 3-cell, “20,  $\times$ ” cage means you need to fill that cage with 3 numbers that multiply to 20.

MysteryGrid 1, 3, 4, 5

4, +		4, $\div$	1, -
20, $\times$	12, +		
			2, -
	15, $\times$		

# MysteryGrid Puzzles

MysteryGrid  $x, 2x, 4x$

$8x^2, \bullet$	$3x, -$	
	$5x, +$	
$4x^2, \bullet$		

MysteryGrid  $a, a^2, a^3, a^4$

$a^6, \bullet$			$2a^4+a^3, +$
$a^7, \bullet$	$a^4, \bullet$		
		$a^5, \bullet$	
	$a^7, \bullet$		

# Puzzling and Persevering

- The I-can-puzzle-it-out disposition
  - Looking for an entry point
  - Building working memory
  - Depending on their own logic
- Puzzle creation: Being *producers*—not just consumers—of mathematics

# Mobles: Collaborative Game

36

Round 1

24

Round 1

40

Round 1

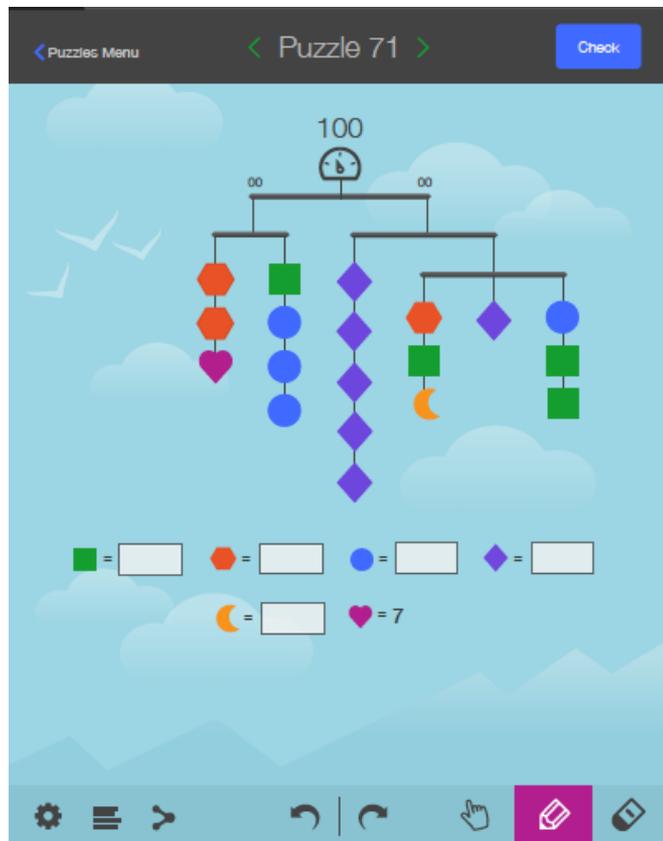
56

Round 1

24

Round 1

# Mobile Puzzle App: SolveMe



Interactive puzzling features:

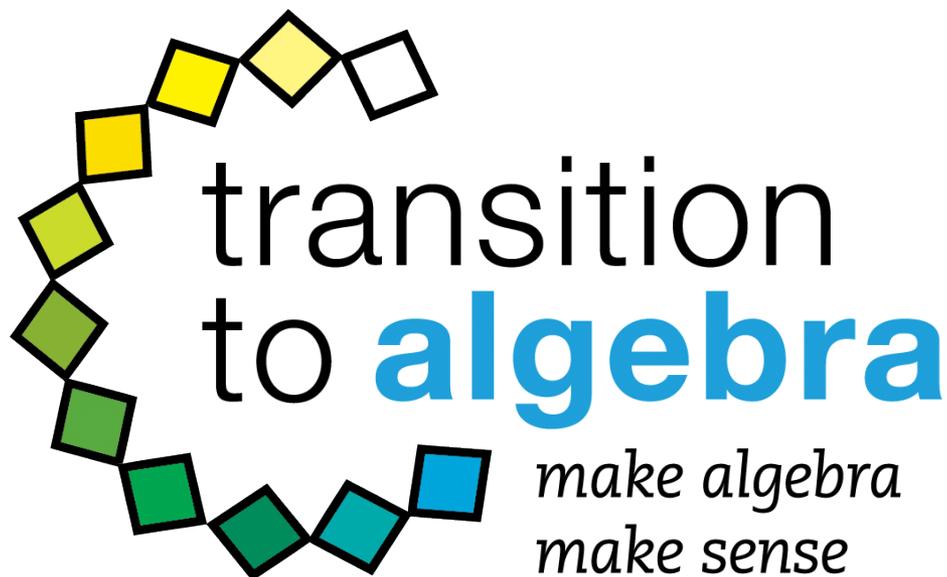
- Shape equations
- Subtraction, division, factoring
- Substitution
- Annotations
- Sharing
- “Build Your Own” mode

[SolveMe.edc.org](http://SolveMe.edc.org)

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# Transition to Algebra



Full-year algebra-support curriculum with student & teacher materials that supports the Common Core Standards for Mathematical Practice

For for information: [transitiontoalgebra.com](http://transitiontoalgebra.com) or [jkang@edc.org](mailto:jkang@edc.org)