How Engaging Puzzles Build Algebraic Habits of Mind

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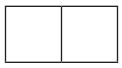
Kentucky Center for Mathematics Conference; Tuesday 3/10 9:25-10:35

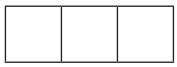
Solving Who Am I? Puzzles

Who Am I?



- The product of my digits is 7.
- The sum of my digits is 8.
- My units digit is greater than my tens digit.

Who Am I?

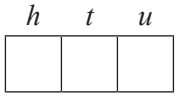


- I am even.
- My tens digit is 1.
- The product of my three digits is 12.
- h is four less than my units digit.

Who Am I?

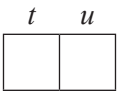
- I am odd.
- $u > t$
- My hundreds digit is prime.
- $t = 2h$
- Two of my digits are square numbers.

Who Am I?



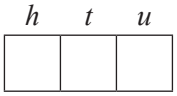
- t is a perfect square.
- $h - u = 0$
- The sum of h and u is 13 more than t.
- I am divisible by 3.

Who Am I?



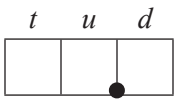
- The product of my digits is 16.
- The sum of my digits is 8.

Who Am I?



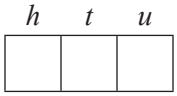
- I am a multiple of 10.
- My hundreds digit is one more than my tens digit.
- The sum of my three digits is 7.

Who Am I?



- I am in the thirties.
- $d < t$
- My units digit is twice my tens digit.
- d is four less than u.
- The product of d and t is u.

Who Am I?



- $u - h = t$
- $3t = h$
- All of my digits are not odd.
- Not all of my digits are odd.
- Not all of my digits are divisible by 3.

transition to **algebra**

make algebra make sense

Curriculum information and presentation documents: **ttalgebra.edc.org**

Sample materials and ordering information: **transitiontoalgebra.com**

SolveMe Mobiles: **solveme.edc.org** and coming soon to iTunes app store

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The material in this handout is based on work supported by the National Science Foundation under Grant No. ESI-0917958. Opinions expressed are those of the authors and not necessarily those of the Foundation.

Who Am I?

n	k	h	t	u

- $t < u$
- $h + k = u$
- I am a multiple of 5.
- k and h are square numbers.
- My only even digit is k.
- I'm less than 75432.
- No two of my digits are the same.
- $t + k = n$

Who Am I?

k	h	t	u

- No two of my digits are the same.
- Three of my digits are powers of 2.
- At least one of my digits is prime.
- The sum of my digits is a perfect square.
- $h + t = k + 2u$
- Three of my digits are perfect squares.
- The difference between t and u is 5.
- $9h = t$

Who Am I?

h	t	u

- $u^2 = h + t$
- $h + t = 2u$
- $h > t$
- $ht > u$

Who Am I?

h	t	u

- I'm odd.
- $h + t + u = 6$
- My units digit > my tens digit.
- $h > t$
- $t + u = h$
- The product of my three digits is 0.

Who Am I?

u	d	c	m

- I am less than 1.63.
- I am greater than 1.6.
- c is one-third of d.
- $m > c^3$

Building Who Am I? Puzzles

Puzzle Building Steps:

1. **Choose the final answer & construct clue boxes.**
2. **Create clues to help identify the answer.**
3. **Check that the clues lead to a unique solution.**

n	k	h	t	u	d	c	m

These variables were selected to match place values and metric system prefixes (n for thousands, k for kilo-, h for hundreds, t for tens, u for units, d for deci-, c for centi-, and m for mili-).

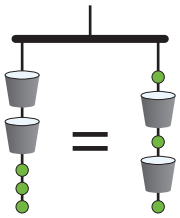
Who Am I?

Who Am I?

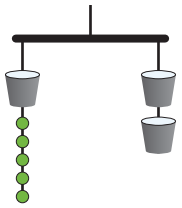
Solving Mobile Puzzles

In each of these problems, a dot (●) equals 1.

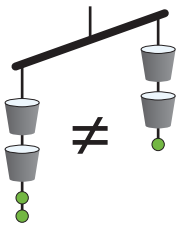
① This mobile *always balances*. Why?



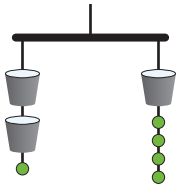
② This mobile *only balances when* the buckets represent a certain number. What number makes it balance?



③ This mobile *never balances* no matter what number the bucket represents. Why?



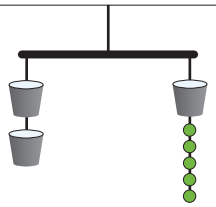
④ Does this mobile balance *always, sometimes,* or *never?*



If sometimes, *when?*

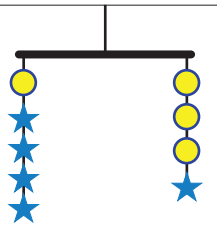
Every beam in the mobiles below is balanced. The strings and the beams weigh nothing. Find the weight of each shape.

⑤



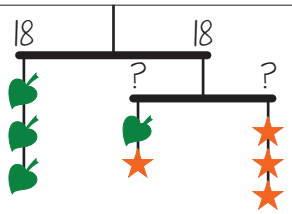
● = 1 ▭ = _____

⑥



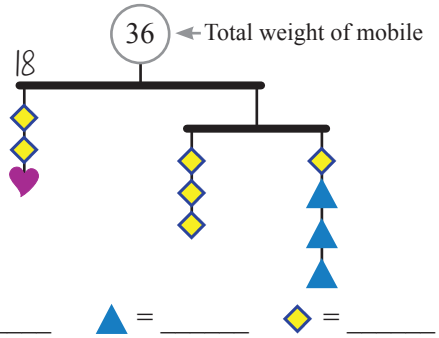
★ = 4 ○ = _____

⑦



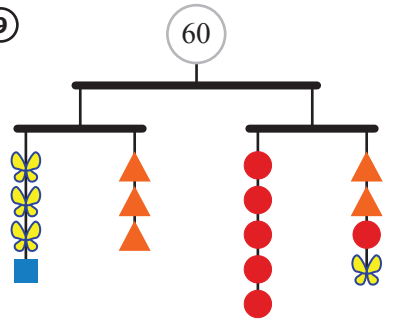
🍃 = 6 ★ = _____

⑧



♥ = _____ ▲ = _____ ◇ = _____

⑨



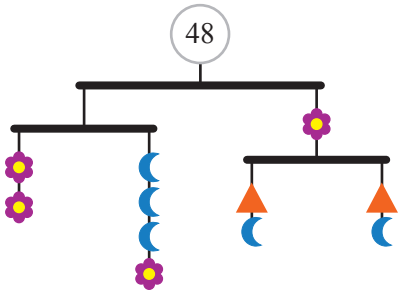
■ = _____

▲ = _____

🦋 = _____

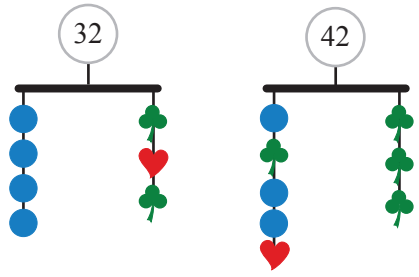
● = _____

⑩



▲ = _____ ☾ = _____ ✿ = _____

⑪



♥ = _____ ● = _____ ♣ = _____

Building Mobile Puzzles

To make a mobile puzzle, start by picking your own shapes and making up the solutions first. Then make up a balanced mobile.

= _____

= _____

= _____

Before you share your mobile, make sure that the solutions you started with are the only possible solutions.

Mystery Number Puzzles

① What are the only two numbers that ● could be if ● • ● = ● ?

● = ____ or ____

② What is the only number that ♥ could be if ♥ + ♥ = ♥ ?

♥ = ____

③ What could ☆, 💧, and ⬢ be if all the variables represent different numbers?

💧 • ⬢ = ☆
⬢ + ⬢ = ☆
💧 + 💧 + 💧 = ☆

☆ = _____
💧 = _____
⬢ = _____

④ What could ⬠ and ♣ be?

⬠ • ⬠ = ♣
⬠ + ⬠ = ♣
⬠ ≠ ♣

⬠ = _____
♣ = _____

⑤ What could ■, ⬠, ♦, ●, and ▲ be if all the shapes are *different* single-digit numbers (0-9)?

■ • ■ = ■
■ + ▲ = ♦
● + ● = ♦
■ + ⬠ = ●
⬠ • ● = ♦

■ = _____
⬠ = _____
♦ = _____
● = _____
▲ = _____

⑥ What could ⬟, ♦, ●, △, and ★ be if all the shapes are *different* single-digit numbers (0-9)?

⬟ • △ = ♦
△ + △ = ♦
⬟ + ⬟ = △
⬟ + △ = ★
⬟ • ● = ★

⬟ = _____
♦ = _____
● = _____
△ = _____
★ = _____

⑦ None of ▲, □, ●, ★, and ⬟ are negative. Use the clues below to figure out the values.

▲ • ⬟ = ⬟
□ • □ = □
● + □ = ▲ • ▲
★ + ★ = ▲
★ • ★ = ▲
□ > ⬟

▲ = _____
□ = _____
● = _____
★ = _____
⬟ = _____

⑧ 🦋, ■, 🍃, ○, and ♥ can be any numbers at all and ○ ≠ ♥. Figure out what they must be.

♥ • ♥ = ♥
♥ + ♥ = ♥
🍃 • ○ = 🦋 • ■
🍃 + ○ = ♥
○ • ○ = ■
○ + ○ = ■

🦋 = _____
■ = _____
🍃 = _____
○ = _____
♥ = _____

Solving MysteryGrid Puzzles

Use the clues to fill in each grid so that every row and every column contains all of the numbers in the title.

5, 7, 9 Latin Square

	5	
7	9	
		9

r*, *s*, *t Latin Square

	s	
		r

MysteryGrid **3, 4, 5**

2,–		7,+
20,•	4	
	8,+	

MysteryGrid **3, 6, 9**

18, •		15, +
54, •	15, +	

MysteryGrid **3, 4, 5, 6**

11, +		36, •	
4	15, •	10, +	
18, •			30, •
	20, •		

MysteryGrid $\frac{1}{3}$, $\frac{1}{2}$, **1**

$1\frac{1}{2}$, +	$\frac{1}{6}$, •	
	$\frac{1}{2}$, –	$\frac{1}{3}$, •
$\frac{1}{3}$		

MysteryGrid **6, 7, 8, 9**

30, +	72, •	63, •	
		30, +	
		48, •	
42, •			

MysteryGrid **1, 2, 3, 4**

8,•		6,•	4,•
4,+			
3,–	5,+	7,+	
		3,+	

MysteryGrid **1, 3, 5, 7**

21, •	7, •		15, +
	25, •		
2, –			
12, +		3, ÷	

MysteryGrid **0.1, 0.2, 0.3, 0.4**

.6,+		.08,×	
	.016,×	3,÷	
.12,×			.5,+
	.02,×		

MysteryGrid a, a^2, a^3

a^4, •	a^2+a, +	
	a^6, •	a^5, •

MysteryGrid $(a-1)$, a, $(a+1)$

$2a-1$, +		a^2+a, •
$2a+1$, +	$a-1$	
	a^2-1, •	

MysteryGrid **(c + 3), (c + 4), (c + 5)**

$c^2 + 8c + 15$, •		$c^2 + 7c + 12$, •
$c^2 + 9c + 20$, •	$2c + 7$, +	
		$c + 5$

MysteryGrid a^{-1}, a, a^2, a^3

a^3, •	a, •	a^6, •	
a^4, •		a^5, •	1, •
a, •			

MysteryGrid **2, 3, 5, 7, 11**

42,•		3	35,•	110,•
	55,•			
27,+	7	12,•		
		23,+		21,•
6,•				

Building MysteryGrid Puzzles

1. Choose a grid size and pick a combination of three or four numbers or expressions with variables. Fill in the grid like a Latin Square puzzle with exactly one of each number or expression in each row and column.

2. Then make your cages.
Block off a group of numbers.
Use an operation (+, –, •, or ÷) to make your clue. These example grids have been started.

MysteryGrid **2, 3, 4, 7**

2	7	9, + 4	3
3	4	7	2
7	2	3	4
4	3	5, − 2	7

MysteryGrid **2x, (x + 2), x^2**

$2x^3$, • $2x$	x^2	$x+2$
x^2	$x+2$	$2x$
$x+2$	$2x$	x^2

You can make cages with just one number, too.

For subtraction and division, use cages with only two numbers.

3. Make sure there is *only one solution*.

Puzzles with more than one solution aren't "wrong," but they aren't satisfying because the player will get stuck at the point where there is no unique answer.

Copy only your clues and try solving your puzzle yourself before sharing it with someone else.

Adjust the cages as needed to make the puzzle have only one solution.

MysteryGrid ___, ___, ___

MysteryGrid ___, ___, ___, ___
